**Structured Hybrid Project Report – Fall 2020**

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# Introduction:

In this project, we will try to price a hybrid derivative contract paying

with

* the STOXX50E spot price quantoed from EUR into USD
* , the = 3-month USD LIBOR rate observed on time T between and T at time t
* , the expiration date
* , given relative strike prices

# Assumptions:

In the given payoff above, the stock price ST is denominated in USD, so we model the stock index price process in terms of USD risk-neutral measure. Note that if the STOXX50E prices where to be quoted in EUR, then price process would simply be a geometric brownian motion in terms of EUR risk neutral measure.

We also assume that foreign interest rate (in this case EUR) is deterministic

Next,

# Methodology:

## Stochastic processes for modelling a hybrid equity index/exchange rates:

The first task here is to describe the stock price process as a GBM in terms of the USD risk neutral measure (this needs to be done as it is originally in the EUR currency). This is described the following process:

where

is the foreign interest rate which is deterministic,

is the correlation of STOXX50E market and USD EUR exchange rate

is the exchange rate vol of USD EUR exchange rate

is the composite vol as elaborated below

## The exchange rate process is given by below:

We model USD-3month LIBOR short rate either using Vasicek or 2-factor Hull-White model:

## Vasicek model:

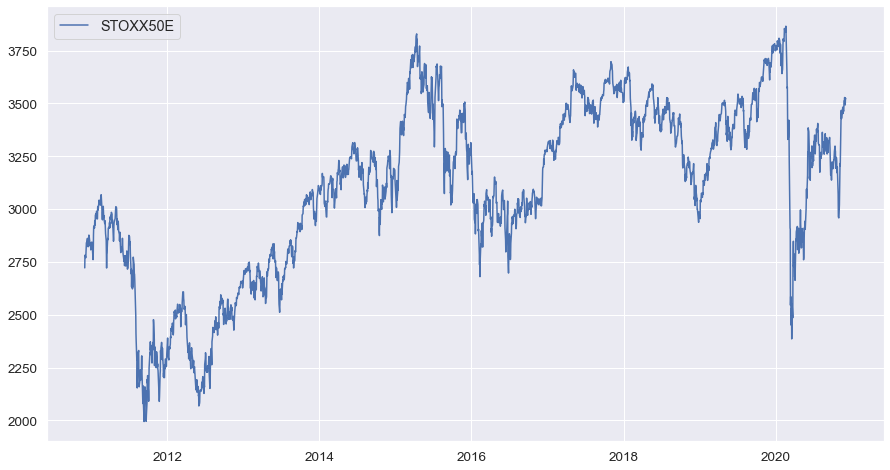
## 2 – factor Hull-White model:

The Hull-white model has a structure similar to Vacisek model but the difference is that is time-dependent variable in the Hull-White model to make it fit the current term structure.

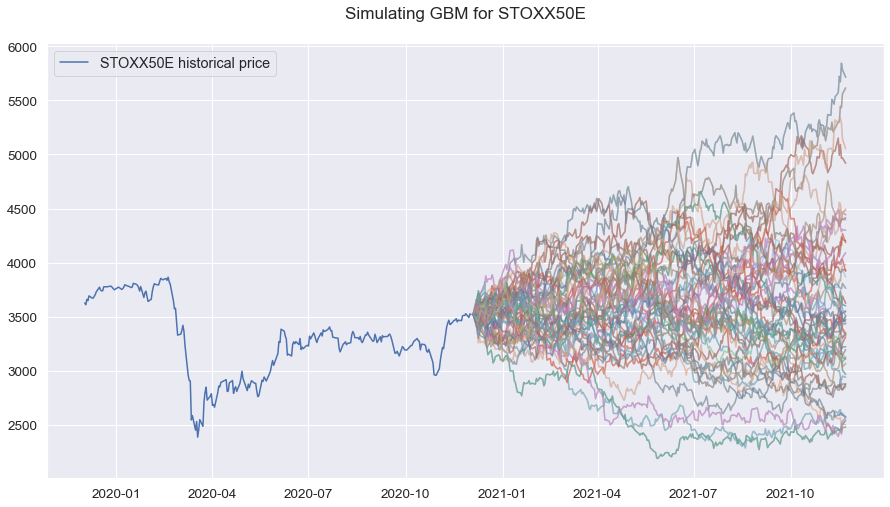
# Model calibration and Simulation:

## Equity/FX model:

We use historical data of EUR/USD rates and STOXX50E prices to calibrate the equity/FX model



Simulated paths of STOXX50E from GBM model:



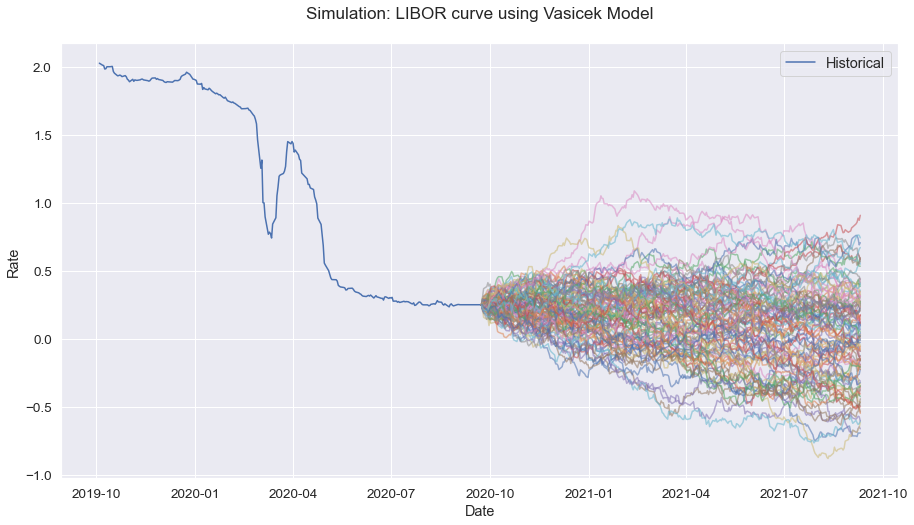
## Rates model:

Vasicek model:

Below is the chart of historical USD 3-month LIBOR rates:



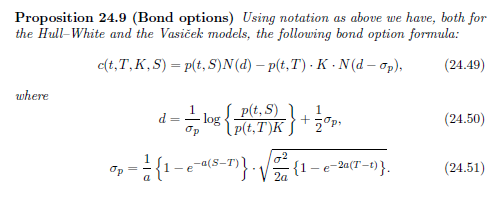
Simulated 3-month USD LIBOR rates:



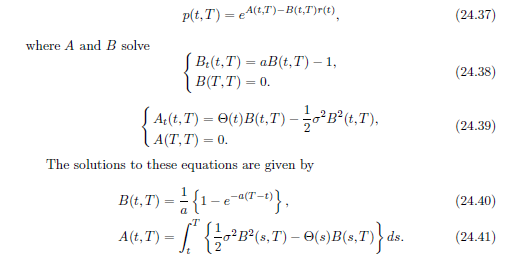
For Hull-White model, the following equations get the closed-form solution of

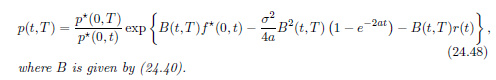


can be determined by the following calibration since we have the historical caplet prices



Now, the bond prices can be determined by the following affine rate model:





Now that we have the bond prices, we can derive the LIBOR rates as follows:

# Conclusion:

Finally, we run Monte Carlo simulation of to obtain price at time of the derivative contract which is given by

Now the pricing engine takes the following inputs from the user to output

1. , current Stoxx50E price quoted in USD
2. , , time to expiration and relative strike prices respectively
3. , current forward USD-Libor 3m rate between and